Indian Statistical Institute, Bangalore Centre

B.Math (Hons) III Year, Second Semester

Backpaper Examination

Comb. and Graph Theory

Time: 3 Hours May 2012 Instructor: N.S.N.Sastry

Total Mark : 100

Note: Answer all questions. Your answers should be complete and clearly written.

1. a) Define a symmetric design. For any prime power $q, q \equiv 3 \pmod 4$, show that there is a symmetric design on $q-1$ points.

   b) Show that there is no symmetric $2-(43,7,1)$ design.

   [4+10+8]

2. a) Define a strongly regular graph. If G is a strongly regular graph with usual parameters v, k, λ, μ, show that $k(k-\lambda-1) = (v-k-1)\mu$.

   [4+8]

   b) If G is a strongly regular graph with $\lambda = 0$ and $\mu = 1$, show that the valency of G is one of $2, 3, 7, 57$. [12]

3. a) Define the following: (i) Perfect code, (ii) Maximum distance separable code (iii) Cyclic code. [4+4+4]

   b) Factorize the polynomial $X^{14} - 1$ over $\mathbb{F}_2$. [8]

4. a) Show that a $t-(v,k,\lambda)$ design $(t > 0, v > k+2, \lambda > 0)$ is also an $s-(v,k,\lambda_s)$ design for each $s \leq t$. Calculate λ_s. [6]

   b) Show that there is no affine plane of order 6. [12]

5. a) Define an extendable design. Give an example of an extendable symmetric design. [8]

   b) Let $I = (X, \mathbb{B})$ be a $3-(n^2+1, n+1, 1))$ design and $x \, \varepsilon \, X$. Then, show that $I_x = (X \backslash \{x\}, \{B \backslash \{x\} : x \, \varepsilon \, B \, \varepsilon \, \mathbb{B}\})$ is an affine plane of order n. [8]

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